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Stochastic Resistance Moment Modeling of Singly Reinforced Concrete Slabs With CFRP

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Simulating the resistance moment of Carbon Fiber Reinforced Plastic (CFRP) in singly reinforced slabs using annealing models was carried out in order to predict resistance moment in these members, this paper presents the research results. Equations for resistance moment were obtained based on the flexural requirements in the ACI 440. 1R-06 code; these equations were then optimized using the Simulated Annealing (SA) algorithm. It was found out that the most important factors which influence the ultimate load carrying capacity of CFRP reinforced concrete structures are; effective depth and breadth of the member, concrete strength and the material properties of the CFRP reinforcement. Thus, the ultimate moment of resistance based on concrete failure obtained for a CFRP singly reinforced concrete section is found to be 22.4% lower than the predicted in the code formulation.

Keywords: Simulation, CFRP, reinforced concrete slabs, resistance moment.

I. Introduction

The use of CFRP internal reinforcements in lieu of conventional steel reinforcements requires a better understanding under various loading and performance conditions (Benmokrane, 2010). This work presents the simulation of the resistance moment of CFRP singly reinforced slabs in flexure using a formulated computer algorithm which finds the optimum value of the objective function, the optimal moment capacity (k_{opt}), as obtained using the nominal moment equations in the ACI 440 R-06, (2006) code. The optimum resistance moment of CFRP reinforced concrete sections M_{opt} is then obtained fromk_{opt}. Unlike other optimization problems, here the objective function is considered as the maximization of resistance moment rather than the reduction of cost. The algorithm developed for the optimization is adopted from the method of Simulated Annealing and it is executed using Visual Basic for Applications, VBA.

The parameters that affect the resistance moment of CFRP reinforced concrete slabs include the flexural strength of members, cross sectional properties, geometric and material properties of reinforcing CFRP bars. Among all these properties, the member's effective depth, *d*, breadth, *b*, concrete compressive strength,$f_c' \text{ or } f_{ck}$, and CFRP bar diameter, d_f, are regarded as the random variables, while the modulus of elasticity of CFRP bars, E_f, is treated as a deterministic design variable in the assessment.

This work aims at improving the understanding of the flexural behavior of CFRP reinforced slabs while obtaining its optimum resistance moment in singly reinforced concrete slabs and sections that can be safely used for their designs. This is considered because,

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asides the well-recognized advantages that CFRP strengthening systems possess, there are also some important doubts using it; bearing in mind that the CFRPs are new materials and not so widely used, there is therefore a need to determine the optimum resistance moment of CFRP in singly reinforced concrete slabs and sections which will satisfy the design requirements of ACI 440 R (1996), ACI-318 (2002), BS 8110 (1985, 1997), CAN/CSA-S6 (2002) and EC2 (2008) as may be necessary, so as to justify its substitution for steel reinforcements in concrete.

Ii. Simulated Annealing Background

Simulated annealing algorithm was derived from statistical mechanics. Kirkpatric et al. (1980) proposed an algorithm which is based on the analogy between the annealing of solids and the problem of solving combinatorial optimization. Annealing is the physical process of heating up a solid and then cooling it down slowly until it crystallizes. The atoms in the material have high energies at high temperature and therefore have more freedom to arrange themselves. As the temperature reduces the atomic energy decreases and a crystal with regular structure is obtained at the state where the system has minimum energy. If the cooling is carried out very quickly, which is known as rapid quenching, wide spread irregularities and defects are seen in the crystal structure, the system does not reach the minimum energy state and ends in poly crystalline state, which has higher energy. At a given temperature, the probability distribution of system energy is determined by the Boltzmann probability as noted in Equation (1).

$$P(E) \propto e^{[-El(kT)]} \qquad (1)$$

where, *E,*is system energy, *k* is Boltzmann's constant , *T* is temperature and *P(E)* is the probability that the system is in a state of energy *E*. At high temperatures, *E* converges to 1 for all energy states according to Equation (1). It can also be seen that there exist a small probability that the system might have high energy even at low temperatures. Therefore, the statistical distribution of energies allows the system to escape from local minimum.

A. Work Done on Optimization of CFRP Reinforced Concrete Structures

Various optimization techniques for reinforced concrete structures have been performed in the past by various researchers but data on the algorithms developed using simulated annealing for optimization of fiber reinforced concrete structural elements are quite limited. Optimization techniques for the element level of reinforced concrete structures have been presented for example in Adamu et al, (1994). These methods were based on sequential linear programming, continuum, that is, type optimality criteria, and nonlinear programming such as Powell's algorithm. Recently the discrete optimization of structures has been performed using Genetic Algorithms (Adamu et al, 1994). Very little literature is available in the field of FRP reinforced concrete structural optimization because design methods for FRP are yet to be fully developed, though some guidelines are available for its applications to airfield pavements and some hydraulic structures (Hwan, 1992). So is the case with standard test procedures to be adopted for testing and evaluation of the performance of FRP elements. Ezeldin and Hsu (1992) optimized reinforced fibrous concrete beams using direct search technique. The algorithm conducts a systematic search in the space of four variables which include beam width, beam depth, fiber content, and aspect ratio of fibers to yield an optimum solution for a given objective function.

The application of an algorithm using simulated annealing that performs a search for the optimum resistance moment of CFRP singly reinforced concrete is presented.

B. Flexural Analysis of CFRP Reinforced Concrete Slabs to Obtain Resistance Moment Equations

The flexural capacity of concrete members reinforced with CFRP bars can be calculated based on assumptions similar to those made for members reinforced with steel bars. Both concrete crushing and CFRP rupture are acceptable failure modes in governing the design of CFRP reinforced concrete slabs provided that strength and serviceability criteria are satisfied. The flexural capacity of a singly reinforced section will be governed by:

(i) Rupture of the CFRP tension reinforcement in which case, the section is referred to as an under-reinforced section.
(ii) Crushing of the concrete where the section is referred to as an over-reinforced section and
(iii) Simultaneous failure of both the concrete and the tension reinforcement, in which case the section is referred to as a balanced section. This case is also referred to as a balanced strain condition.

The assumptions for design of a singly reinforced concrete section in the ACI code are given as follows (ACI 440.1R (2006)):

(1) A plane section before loading remains plane after loading.
(2) The maximum usable compressive strain in the concrete is assumed to be 0.003mm/mm.
(3) The tensile strength of the concrete is ignored.
(4) The tensile behavior of the FRP reinforcement is linearly elastic until failure.
(5) Perfect bond exists between concrete and FRP reinforcement.
(6) A rectangular stress block is used for concrete in compression.
(7) The compressive force equals the tensile force.
(8) The internal moment equals the applied bending moment.

The analysis presented herein is based on the conventional compatibility and equilibrium conditions used for normal reinforced concrete except that the material properties of CFRP are put into consideration.

Long term exposure to various types of environments can reduce the tensile strength, creep rupture, and fatigue endurance of CFRP bars, hence, material properties used in design equations should be reduced based on the type and level of environmental exposure. The design tensile strength should be determined by Equation (2) (ACI – 440-1R (2006);

$$f_{fu} = C_E f_{fu}^{*} \qquad (2)$$

where f^{*}_{fu} is guaranteed tensile strength of CFRP bar, and f_{fu} is the design tensile strength of CFRP (ACI 440.1R (2006)). The Equation (3) given below indicates the safety criterion for design of reinforced concrete sections (ACI- 318 (2002), that is;

$$M_u \leq \emptyset M_n \quad (3)$$

where, $M_u =$ Factored ultimate moment based on applied loads times load factors; $M_n =$ Nominal flexural strength and $\emptyset =$ Strength reduction factor.
The failure mode can be determined by comparing the CFRP reinforcement ratio, ρ_f to the balanced reinforcement ratio, ρ_{fb}, which is the ratio where concrete crushing and CFRP rupture occur simultaneously.

The balanced failure condition is defined as the situation, where strains in concrete and CFRP bars simultaneously reach their predefined limiting values, i.e., $\varepsilon_c = 0.003$ and $\varepsilon_f = \frac{f_f}{E_f}$, in concrete and CFRP bars, respectively. Although the balanced failure condition is difficult to achieve in practice, the concept of balanced failure helps in defining the tension and compression failure modes. The CFRP reinforcement ratio, ρ_f, to the amount of concrete in a section is given as (ACI 440 – 1R (2006):

Iii Basic Elements of the Simulation

When a combinatorial optimization problem and an annealing process are compared, the states of solid represent feasible solutions of the optimization problem, the energies of the states correspond to the values of the objective function computed at those solutions, the minimum energy state corresponds to the optimal solution to the problem and the rapid quenching can be viewed as local optimization.

The algorithm consists of a sequence of iterations. Each iteration consists of randomly changing the current solution to create a new solution in the neighborhood of the current solution. The neighborhood is defined by the choice of the generation mechanism. Once a new solution is created, the corresponding change in the objective function is computed to decide whether the newly produced solution can be accepted as the current solution. If the change in the objective function is negative the newly produced solution is taken as negative, otherwise, it is accepted according to Metropoli's criterion based on Boltzmann's probability as stated in Equation (1).

According to Metropoli's criterion, if the difference between the objective function values of the current and the newly produced solutions is equal to or larger than zero, a random number*(δ)* in [0,1] is generated from a uniform distribution and if

$$(\delta) \leq e^{-\frac{\Delta E}{T}} \quad (4)$$

then the newly produced solution is accepted as the current solution. If not the current solution is unchanged as expressed in Equation 4. Δ*E* is the difference between the objective function values of the two solutions.

In order to implement the algorithm for a problem, there are four principal choices that must be made. These are: (i) representation of the solutions; (ii) representation of the objective function; (iii) defining the generation mechanism for the neighbors and (iv) designing a cooling schedule.

Solution representation and cost function definitions are the same as those for genetic algorithms. Different generation mechanisms could be developed which could also be borrowed from genetic algorithms, for example mutation and inversion.

Also, in order to design the cooling schedule for a simulated annealing algorithm, four parameters that are specified are: an initial temperature, a temperature update rule, the number of iterations to be performed at each temperature step and a stopping criterion for the search.

Several cooling schedules are adopted in this work and they employ different temperature updating schemes, of these, stepwise, continuous and non-monotonic temperature reduction schemes include very simple cooling strategies. An example is the geometric cooling rule. This rule updates the temperature by the following formula

$$T_{i+1} = cT_i i = 0,1 \dots.. \tag{5}$$

where c is a temperature factor which is a constant smaller than 1 but close to 1.

In deriving the objective function, some transformations are needed because simulated annealing is ideally suited for unconstrained minimization optimization problems but the present problem is a constrained maximization problem, these are explained below. When the constrained maximization is to be transformed into an unconstrained minimization, two transformations need to be made. The first transformation transforms the original constrained problem into an unconstrained problem, using the penalty function concept. A formulation based on the application of penalty whenever there is a violation of specified constraints, is used for this transformation. If the design variable set violates the constraint then a lower value of say 1.0 is assigned and if not, a higher value of say 10.0 is assigned as violation parameter. The violation coefficient, φ , can be computed as follows:

$$\varphi = \sum_{i=1}^{m} \varphi_i \tag{6}$$

$$\varphi_i = a.g_i(x) \tag{7}$$

$$g_i(x) < 0, a = 10; \ and \ \ g_i(x) \geq 0, a = 1.0$$

$m =$ number of constraints
$a =$ penalty parameter
$g_i(x) =$ constraint function

$$Z_m(x) = Z(x) + \varphi \tag{8}$$

where $Z(x)$ is the objective function subject to:

$X_i^l \leq X_i \leq X_i^u \qquad i = 1, 2 \dots., n$ (9)

and

$X_i^l =$ Lower limit and X_i^u =Upper limit

The objective function (k_{opt}) for this work is the maximization of moment capacity *(k)* of CFRP reinforced concrete slabs. Therefore, if;
If,

$$k_{opt} = maximise \left[\frac{M_u}{bd^2 f_{cu}}\right] \qquad (10)$$

Then the optimized ultimate moment is given by

$$M_u = k_{opt} bd^2 fcu \qquad (11)$$

The second transformation involves transforming the maximization problem into a minimization problem. The maximization of a function *Z(x)* is equivalent to the minimization of the negative of the same function. For example, the objective function: Minimize *Z(x)* is equivalent to
Maximize $Z'_m(x) = -Z_m(x)$ (12)

A. Forming the Constraints

The following constraints must be formed based on conditions presentend in the various necessary codes.

(i) *Constraints on Ultimate Moment*

(ii) *Constraint for Limiting Span/Depth Ratio for Deflection Control*

(iii) *Constraints on Design Variables*

(iv) *Stress in the CFRP*

B. Running the Algorithm

The algorithm which is temperature dependent starts with a high temperature,T_0, a sequence of design vectors is then generated randomly until equilibrium is reached; that is, the average value of 'Z_m.' reaches a stable value as '*i*' increases. The best point reached is recorded as X_{opt}. Once thermal equilibrium is reached, the temperature *T* is reduced and a new sequence of moves is made and continued until a sufficiently low temperature is reached, at this stage no more improvement in the objective function value can be expected. The basic algorithm is shown as a flow diagram in Figure 1.

The algorithm starting from an initial vector, X_1, then generates successively improved points randomly as X_2,X_3,...., and moving towards the global maximum solution. If X_i, denotes the current point, random moves are made along each coordinate direction, in turn. The new coordinate values are uniformly distributed around the corresponding coordinate of X_i,. One half of these intervals along the coordinates are stored as the step vector, S_i. If the point falls outside the range given in Equation 9 a new point satisfying Equation (9) is found. A candidate design vector X is accepted or rejected according to a criterion known as the Metropolis criterion (Equation (4)).

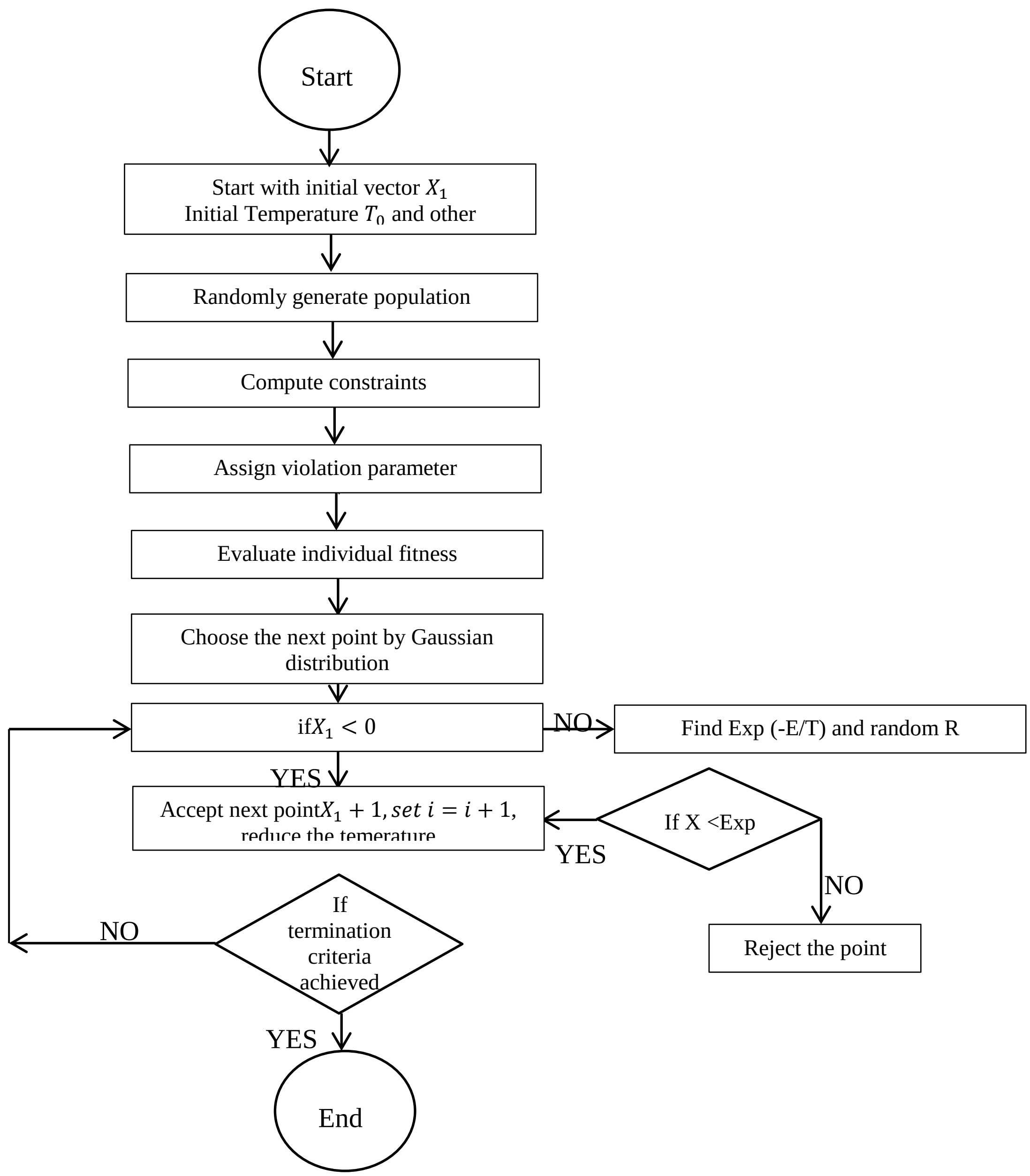

Figure 1: Flow chart for the algorithm

Stopping conditions are defined in the algorithm because if the simulated annealing is terminated too early, a suboptimal solution will be obtained, while if it is terminated too late, the extra iterations take up computational time. There are a number of stopping conditions which include (Ingber, 1995): (a) when a maximum number of iterations have been exceeded; (b) when no improvement occurs over a number of iterations; (c) when a certain percentage of the simulated annealing is around an optimum and (d) when the objective function slope is approximately zero.

Iv. Analysis and Results

The concrete structure analyzed in the research is a CFRP singly reinforced concrete slab section with assumed range of geometry. Models for the resistance moment of the slab are obtained from ACI-440-1R (2006) code and modeled using Simulated Annealing (SA) algorithm. The developed algorithm is implemented using Visual Basic for Applications (VBA) and runs on a Microsoft Windows Operating System.

A. Input Parameters

The following are the assumed values of various parameters used in this work. The values are obtained from suggested data in researches and known properties of CFRP and concrete from experience. These are stated as follows: (i) Tensile strength of CFRP = $2068.42 N/mm^2$; (ii) Tensile strain of CFRP $= 0.5$; (iii) Interfacial bond stress between fiber and the matrix (τ) =7.0 N/mm^2; (iv) Shear span $a_s = 0.84\, m$; (v) Split tensile strength$(f_t) = 0.1 f_c'$; (vi) Stress value corresponding to failure strain of matrix $\left(f_f\right) = 1.412\, f_{fu}$and (vii) Elastic modulus of CFRP,$E_f = 1.5 \times 10^5 N/mm^2$

The first interface when using the developed simulator is used for inputting parameters which define the CFRP properties, slab properties and the optimization variables which include the upper and lower bounds. These parameters define the objective function to the program codes so that searching space and time are limited to the case study. Each of the input boxes is linked to the VBA code and sends the values inputted by the user to the codes. The command button “Start Simulation” activates the codes, while the command button “Exit”, terminates the code at any point when it is running.

The various simulated annealing parameters used are as follows: (a) Starting value of Temperature (T_0) = 500; (b) No of iterations (niter) = 100; (c) Temperature reducing rate (c) = 0.99 and (d) No of variables (nvar) = 4.

The starting temperature is taking as 500 because during the annealing process, the time spent at each temperature level must be sufficiently long to allow the system to reach a thermal equilibrium or a steady state. If care is not taken in adhering to the annealing temperature schedule, undesirable random fluctuations may cause the shift of the ground state. The basic idea of statistical mechanics initiates a generalization of the iterative improvements or the search for a better solution of the combinatorial optimization. This process is encountered to a ‘Steepest-Descent’ algorithm for Minimization problems or a ‘Steepest-Ascent’ algorithm for maximization problems, in this case the temperature reducing rate ‘c’ is taken as 0.99.

The number of iterations is taken as 1000 because it is expected that convergence will be achieved before the 1000th iteration when the objective function converges to a steady and maximum value. The developed application allows the number of iterations to be set at 100,000 but the minimum which is 1000 is used in the simulation.Also, the number of variables (nvar) is taken as 4, since the program searches in a space of 4 variables, width of slab *(b),* effective depth of slab *(d),* diameter of CFRP, (d_f) and compressive strength of concrete $f_c'(kN/mm^2)$, this also limits the searching time of the program during the simulation.

The second interface is used for inputting parameters which define the annealing starting value of temperature (T_0), number of iterations (niter), temperature reducing rate (*c*) and number of variables (nvar), these parameters guide the annealing part. The width of slab is defined to the model as 1000mm as taken for a unit strip, effective depth of slab is taken in a range between 50 to 200mm as most practical slabs are designed in this range. Diameter of CFRP is taken in the range between 4 to 22mm because this is the range of available CFRP bars. Concrete compressive strength is taken between 20 to 50 kN/mm^2.The parametric study results of the algorithm are shown in Table 1. It is seen that the final values of objective function (given by $k_{opt} = maximise\left[\frac{M_u}{bd^2 f_{cu}}\right]$), improves from the initial stage (0.125694) to the final one (0.194578) with increase in the number of iterations. For each iteration number, the program randomly selects values for the effective depth, concrete strength and CFRP diameter and then it proceeds to calculate the reinforcement ratio and CFRP area.

After some comparison on which should govern the failure type, it calculates the moment capacity and outputs it in the moment capacity column. The program runs until it finds a steady maximum value for the resistance moment capacity when it reaches the maximum number of iterations before stopping. It can be seen from Table 1, that the effective depth of the slab and the diameter of CFRP have a greater influence on the moment capacity than the concrete strength. At a CFRP diameter of 17mm, the section failed by CFRP rupture but at a higher diameter, failure was by concrete crushing.

At the initial stage the variation of the objective function is too high. As the temperature parameter reduces, the variation is also reduced and convergence is achieved at the culminating stage, hence the objective function converges to a steady and maximum value. The objective function for optimization is the maximization of ultimate moment capacity of a singly reinforced CFRP concrete slab subjected to bending. Equations have been formulated and modeled based on the ACI 440 1R-06 code and constraints applied to requirements of necessary codes. The program is designed to read the required data that is, limiting values of variables, and simulated annealing parameters. The program then searches for a maximum for the objective function in the space of the four variables’ limiting values. The maximum of the objective function is recorded if all the constraints are satisfied. If any of the constraint is violated, the penalty for violation is given and the objective function modified by the penalty factor as defined by the user. The modified objective function incorporating the constraint violation is given in Equation (17).

The simulation resulted in an optimal value of **0.194** for the moment capacity of the assumed CFRP singly reinforced concrete slab section. This value when replaced for k_{opt} in the objective function, $k_{opt} = \text{maximise} \left[\frac{M_u}{bd^2 f_{cu}}\right]$ gives the optimum resistance moment (M_{opt}) of CFRP singly reinforced concrete sections as;

$$M_u = 0.194bd^2 f_{cu}. \quad (13)$$

Table 1: Optimal Moment Capacity Results Using Simulated Annealing

Iteration No.	Effect. Depth *(mm)*	Slab Width *(mm)*	Concrete Strength	CFRP Dia(mm)	Reinf. Ratio	CFRP Area (mm^2)	Ultimate Moment $\times 10^6$ KNm	Moment Capacity	Failure Mode
100	142.0	1000	26	18	0.0017	254.502	65.897	0.125694	Conc. Crushing
200	150.5	1000	25	17	0.0015	227.009	70.485	0.124475	CFRP Rupture
300	135.0	1000	24	22	0.0028	380.182	68.494	0.156594	Conc. Crushing
400	137.0	1000	24	22	0.0027	380.182	70.141	0.155711	Conc. Crushing
500	105.0	1000	20	22	0.0036	380.182	41.251	0.187079	Conc. Crushing
600	109.5	1000	20	21	0.0031	346.406	42.718	0.178135	Conc. Crushing
700	105.0	1000	20	22	0.0036	380.182	41.251	0.187079	Conc. Crushing
800	94.0	1000	20	22	0.0040	380.182	34.386	0.194578	Conc. Crushing
900	96.0	1000	20	22	0.0040	380.182	35.600	0.194578	Conc. Crushing
1000	94.0	1000	20	22	0.0040	380.182	34.386	0.194578	Conc. Crushing

V. Conclusion

The primary aim of this work is to simulate the resistance moment of CFRP reinforced concrete slabs using annealing models in order to extend the existing knowledge of slabs with fiber reinforced polymers as the internal reinforcement. From the analytical studies and formulations for the Simulated Annealing-based optimal moment of resistance of CFRP concrete slabs along with identification of the objective function, constraints and design variables, the following conclusions are made.

1. This study resulted in an optimal value of **0.194** as the moment capacity of CFRP slabs, despite the fact that the durability of CFRP reinforcing bars for concrete application are not extensively researched and not well understood.
2. This is about ….% higher than the formulated for steel reinforced single concrete slabs.
3. The optimum design results can well be controlled by the designer by specifying various design requirements.
4. With the results from this study, the optimal resistance moment of CFRP reinforced concrete slabs can be well understood since every single step of the optimization process was reported accurately in a well formatted manner and respective details indicated..

5. Limited research data are available on creep characteristics of concrete structural elements reinforced with CFRP bars, though such data are extensively available for slabs reinforced with steel bars.

6. Comparison of the failure modes based on results from running the developed simulated annealing based application has indicated better member deformability (analogous to ductility in steel reinforced concrete slabs) and gradual member failure in compression as compared to tension failure.

7. Higher moment of resistance was observed in compression failures (concrete crushing) as compared to a tension failure (CFRP rupture).

8. The current ACI 440.1R (2006) equations for nominal moment of reinforced concrete sections reinforced with CFRP can be modified with the findings in this work.

9. The ultimate moment of resistance of slabs increases with the amount of CFRP rebars. This is true, obviously, for the case in which the failure mode is compression controlled (crushing of the concrete); this confirms the results of Ezeldin (1992).

10. The developed simulated annealing successfully led the randomly distributed initial design points in the design space to the local optimum design point, hence providing an excellent convergence of the maximization process.

11. In the present approach, the number of the variables used in simulated annealing is considerably reduced as the quantity, area of stirrups, and spacing of stirrups are not considered as variables. They are represented in the algorithm as implicit variables.

12. The efficiency of the algorithm suggests its immediate application to CFRP reinforced concrete and other design optimization problems.

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